# Tracking control

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## Abstract

In this lecture, we introduce and investigate the concept of tracking controllability, where the goal is to ensure that the state of a control system, or a specified projection of it, follows a predetermined trajectory within a given time horizon. Through the use of duality principles, we characterize tracking controllability via a novel observability inequality for the adjoint system. This characterization enables the systematic construction of minimal-norm control inputs. However, establishing this observability inequality presents significant challenges. We first address the problem for the wave equation and then extend the approach to the heat equation using transmutation techniques. Additionally, we examine the implications of tracking controllability within finite-dimensional control systems, highlighting both theoretical insights and practical considerations.

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**Keywords**: tracking controllability, duality principle, observability inequality, transmutation technique, minimal-norm control.

# References

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