Optimization on manifolds for structured matrix problems with fixed eigenvalues

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Abstract

The talk will consider several manifold optimization schemes to solve inverse structured symmetric matrix problems with prescribed spectrum. Some entries in the desired matrix are assigned in advance and cannot be altered, and some others should be nonzero. The rest of the entries are free. The reconstructed matrix must satisfy these requirements and its eigenvalues must be the given ones. This inverse eigenvalue problem is related to the problem of determining the graph, with weights on the undirected edges, of the matrix associated with its sparse pattern. Our optimization schemes are based on considering the eigenvector matrix as the only unknown and moving iteratively on the manifold of orthogonal matrices, forcing the additional structural requirements through a change of variables and a convenient differentiable objective function in the space of square matrices. We propose and analyze Riemannian gradient-type methods combined with either a penalization or an augmented Lagrangian strategy. We also present a block alternating technique that takes advantage of a proper separation of variables. We present some numerical results to demonstrate the effectiveness of our proposals.

This is a joint work with Jean-Paul Chehab and Harry Oviedo.

Keywords: Riemannian optimization, Stiefel manifold, inverse eigenvalue problems, spectral graph theory, augmented Lagrangian.