A Journey on Subspace Methods for the Solution of Sequences of Linear Systems

Luc Giraud[1], Emmanuel Agullo[1], Olivier Coulaud[1], Martina Iannacito[2], Gilles Marait[1], Yanfei Xiang[1]

[1] *Inria, Concace joint team with Airbus CR& T and Cerfacs, firstName.lastName@inria.fr*
[2] *University of Bologna; martina.iannacito@unibo.it*

Abstract In various scientific and industrial applications, solving sequences of linear systems for multiple left and right-hand sides provided either simultaneously or sequentially is a common and challenging task. This talk delves into subspace methods designed to address these challenges, presenting advances that facilitate the efficient solution of such systems in high-dimensional settings.

The first part of the presentation will focus on a class of block methods that allow simultaneous handling of multiple right-hand sides. These techniques face unique challenges, such as managing varying convergence rates across different right-hand sides, necessitating dynamic strategies for adapting the solution subspaces.

In the second part, we will explore tensor-based methods, an essential approach for mitigating the curse of dimensionality. By adapting inexact GMRES with low-rank tensor approximations, we show how these methods make high-dimensional problem-solving feasible. We will also discuss theoretical bounds that evaluate solution accuracy in variable precision settings, particularly when low-rank approximations are employed.

Keywords: block subspace emthods, spectral recycling, subspace extension policies, low rank tensors.

References

[1] Emmanuel Agullo, Luc Giraud, Yan-Fei Jing, Block GMRES method with inexact breakdowns and deflated restarting SIAM Journal on Matrix Analysis and Applications, 2014, 35 (4), pp.1625-1651.

[2] Emmanuel Agullo, Olivier Coulaud, Luc Giraud, Martina Iannacito, Gilles Marait, Nick Schenkels The backward stable variants of GMRES in variable accuracy RR-9483, Inria. 2022, pp.1-77

[3] Luc Giraud, Yan-Fei Jing, Yanfei Xiang A block minimum residual norm subspace solver with partial convergence management for sequences of linear systems SIAM Journal on Matrix Analysis and Applications, 2022, 43 (2), pp.710-739.

[4] Olivier Coulaud, Luc Giraud, Martina Iannacito A robust GMRES algorithm in Tensor Train format RR-9484, Inria. 2022, pp.1-48

[5] Mickael Robbé and Miloud Sadkane, Exact and inexact breakdowns in the block GMRES method, Linear Algebra Appl., 419 (2006), pp. 265–285.