Updating the GLT analysis, new tools, and beyond

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Abstract The class of Generalized Locally Toeplitz (GLT) sequences has been introduced as a generalization both of classical Toeplitz sequences and of variable coefficient differential operators and, for every sequence of the class, it has been demonstrated that it is possible to give a rigorous description of the asymptotic spectrum in terms of a function (the symbol) that can be easily identified.

This generalizes the notion of a symbol for differential operators (discrete and continuous) or for Toeplitz sequences, where for the latter it is identified through the Fourier coefficients and is related to the classical Fourier Analysis.

The GLT class has nice algebraic properties and indeed it has been proven that it is stable under linear combinations, products, and inversion when the sequence which is inverted shows a sparsely vanishing symbol (sparsely vanishing symbol = a symbol which vanishes at most in a set of zero Lebesgue measure). Furthermore, the GLT class virtually includes any approximation of partial differential equations (PDEs), fractional differential equations (FDEs), integro-differential equations (IDEs) by local methods (Finite Difference, Finite Element, Isogeometric Analysis etc) and, based on this, we demonstrate that our results on GLT sequences can be used in a PDE/FDE/IDE setting in various directions, including multi-iterative solvers (combining preconditioned Krylov, multigrid, etc), spectral detection of branches, fast 'matrix-less' computation of eigenvalues, stability issues. We will discuss also the impact and the further potential of the theory with special attention to new tools and to new directions as those based on symmetrization tricks, on the extra-dimensional approach, and on blocking structures/operations.

Keywords:GLT, (g)acs, blocking, flipping, extra-dimensional, approximated PDEs, FDEs, IDEs, eigenvalues, singular values, Weyl distributions, Krylov methods, multigrid, multi-iterative solvers

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